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Reply by Author to A. R. Wadia

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I WISH to thank A. R. Wadia for his interest and remarks concerning my Note.[1] In general, I am in agreement with the ideas expressed in his Comment, but I should like to add the following discussion and to answer the questions posed.

The definition of the Navier-Stokes region used in Ref. 1 is, as Wadia points out, restrictive in that it presupposes that flow reversal does not occur outside of the cavity. If this is true, then addition of the interaction with the third layer—the potential outer flow—is usually sufficient to account for the upstream propagation of disturbances through the subsonic portion of the boundary layer as well as to calculate the correct downstream behavior of the disturbed layer. Such a method was described in Ref. 2 and its use illustrated in Refs. 2 and 3. For the relatively low injection rates into the supersonic boundary layer in Ref. 1 ($v/U_\infty = 0.02$ for the outer flow of $M_\infty = 2.25$) the two-layer approach, as well as the simple choice of the rectangular Navier-Stokes region, were found appropriate for the purposes of that paper. The disturbed nature of the boundary layer downstream of the slot is evident nevertheless in the results obtained in Ref. 1. The qualitatively correct behavior of reduced shear downstream of the injection slot, and increased shear downstream of the suction slot, was reproduced. The computed values of the u velocity component is one grid interval above the horizontal wall (which is linked to $\partial u/\partial y$ at the wall) normalized by the corresponding value of this parameter upstream of the slot are persented in Table 1 for three x steps, starting at the downstream corner, and are intended for qualitative comparison only.

It was shown in the Comment that for a subsonic flow with a relatively high rate of injection ($v/U_\infty = 0.25$) a separation bubble can form downstream of the injection slot. To treat such a flowfield adequately using the interactive method of Refs. 1-4, the Navier-Stokes domain should be extended downstream (and perhaps upstream) of the slot to include at least all the regions of flow reversal, as Wadia recognized. If the incompressible equations are to be retained in the recirculation region and the outer flow is supersonic or highly subsonic so that the incompressibility assumption fails in a significant part of the boundary layer, then the matching region may no longer be specified a priori using a simple, rectangular geometry. Instead, curved matching boundaries, which can evolve as the solution proceeds, are necessary. Such a self-adaptive approach to the construction of the matching region is, at present, beyond the capability of the model, but is a natural subject for further work in extending the method.

The vorticity at the two convex corners of the slot was determined along the vertical walls for Navier-Stokes region and along the horizontal wall for the boundary-layer computations.[4] This discontinuous wall value treatment[5] follows naturally from the definition of the two computational regions and is, in effect, the same as the method used by Wadia and Booth.[6] As the computational step size decreases, the corner becomes "sharper" numerically and the solution may be locally affected. For similar problems, it was found[3] that Δx variations of approximately one order of magnitude (while Δy was held constant) affected the stagnation point location by a fraction of Δy.

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Table 1 Normalized u-velocity component downwind of the slot

Mode	Suction					
Slot width	Narrow slot ($L_s = 0.004\, L_{BL}$)			Wide slot ($L_s = 0.008\, L_{BL}$)		
Distance from corner	C	Δx	$2\Delta x$	C	Δx	$2\Delta x$
$\left.\frac{u}{u_0}\right\vert_{\Delta y}$	1.98	1.54	1.55	2.84	1.75	1.90
Mode	Injection					
Slot width	Narrow slot ($L_s = 0.004\, L_{BL}$)			Wide slot ($L_s = 0.008\, L_{BL}$)		
Distance from corner	C	Δx	$2\Delta x$	C	Δx	$2\Delta x$
$\left.\frac{u}{u_0}\right\vert_{\Delta y}$	1.14	0.38	0.79	0.28	0.23	0.37

Both the centered and upwind differencing were tried for the convective terms in the numerical model. Because the cavity Reynolds numbers were low [0(10)] the results generated by both schemes were virtually identical. No convergence problems were encountered with either method in any of the cases examined. The primary constraint on applicability of the present approach is the selection of the matching boundaries.

The remaining details are now addressed. No closed recirculation regions were seen within the slot for the cases with injection (although separation beneath the upstream corner did appear for the narrow slot in Fig. 3a of Ref. 1).

The depth of the slot was 1.25 L_s and 2.5 L_s, respectively, for the wide and the narrow configurations. An initial boundary layer profile of the Blasius form was specified 25 narrow slot lengths ahead of the interaction.

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